Starter Questions

The graph of $y = x^2 - \frac{1}{3}x^3 + ax$ passes through (3,24).
Find the x- coordinates of the stationary points

A curve has $\frac{dy}{dx} = 3x^2 + 6x + 2$ and passes through the point (0,-1). Find the equation of the curve.

Starter Questions

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$x = 3 \quad y = 24 \qquad 9 - \frac{1}{3} \times 27 + 3a = 24 \qquad a = 8$

$$\frac{dy}{dx} = 2x - x^2 + 8$$

$$\frac{dy}{dx} = 0 \quad 2x - x^2 + 8 = 0$$

x = -2 x = 4

Starter Questions

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H Integration

H3

Evaluate definite integrals; use a definite integral to find the area under a curve and the area between two curves.

Students should be able to:

- understand and use the fact that for a function, f, where $f(x) \geq 0$ for $a \leq x \leq b$ the area between the x-axis, the curve $y = f(x)$ and the lines $x = a$ and $x = b$ is given by

$$\text{area} = \int_a^b f(x)\,dx$$

- understand that for areas lying **below** the x-axis the definite integral will give the negative of the required value
- find areas between curves and straight lines.

Notes

- Definite integrals can be found on a calculator and students are expected to do this in exams. If exact answers are required, these will usually require a non-calculator method.
- Students are **not** expected to find an area between a curve and the y-axis, by integrating an expression for x with respect to y

4.7 Area under a curve

So far, we have seen indefinite integration where the integral produces a function. A definite integral produces a value.

A definite integral is denoted by:

$$\int_{a}^{b} f(x)\,dx$$

where and are the **limits of integration** - is the lower limit and is the upper limit.

4.7 Area under a curve

The limits of integration tell the range of -values to integrate the function between.

To work out a definite integral, we integrate as normal, substitute in the limits and subtract the results. As we are subtracting, we do not need to include the constant of integration as these would cancel out.

If is the derivative of for all values of in the interval then the definite integral is defined as:

This relationship is known as the **fundamental theorem of calculus**.

4.7 Area under a curve

mple 1: Find the value of these definite integ

$$\int_1^3 10x^4 \, dx$$

$$\int_2^3 8x^3 + \ 2x \, dx$$

You can also do these on your calculator! Look for the integration symbol and try it.

H

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4.7 Area under a curve

When we are asked to find an exact value of a definite integral, we must complete the integration ourselves as the calculator will not give answers in exact form.

ple 2: Find the exact value of this integr

Tuesday, May 27, 2025

4.7 Area under a curve

mple 3: Find the possible values of

$$\int_1^5 2px+7\,dx=4p^2$$

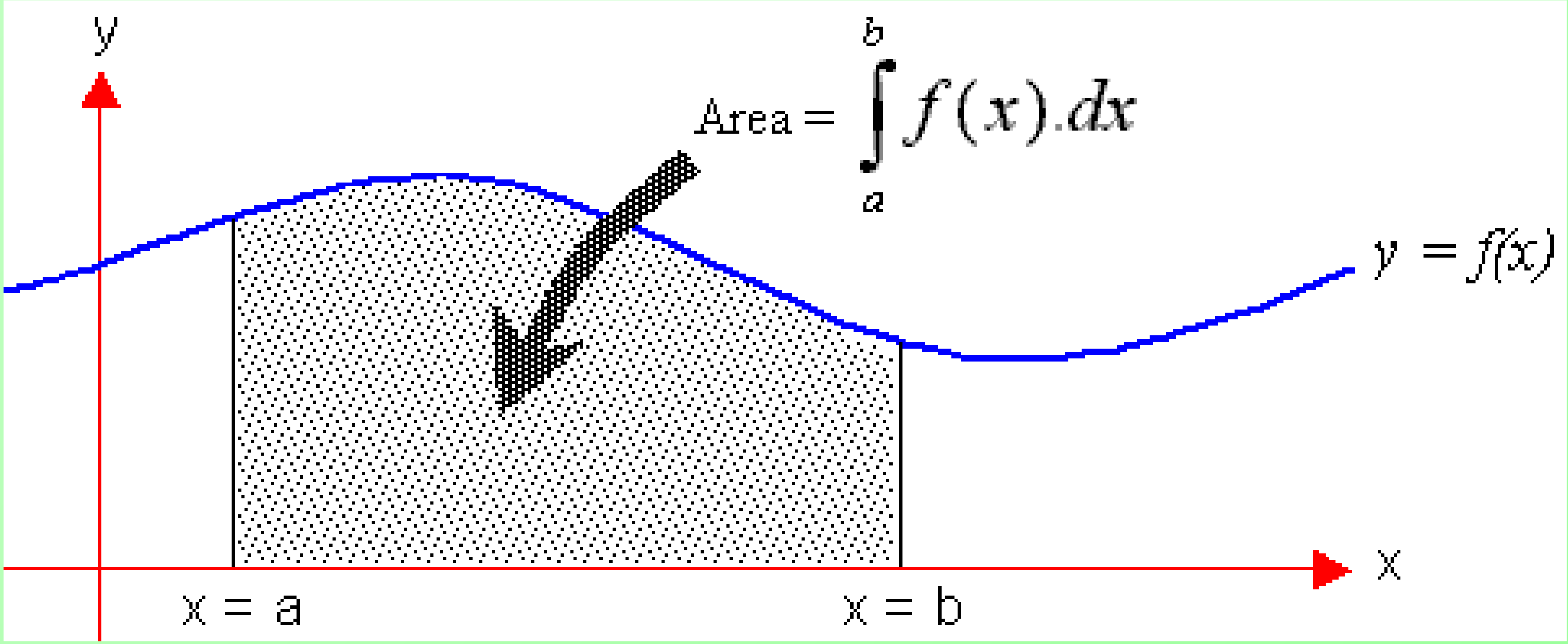
Definite Integrals Worksheet:

Q1-3 check on your calculator

Q4-8 do these

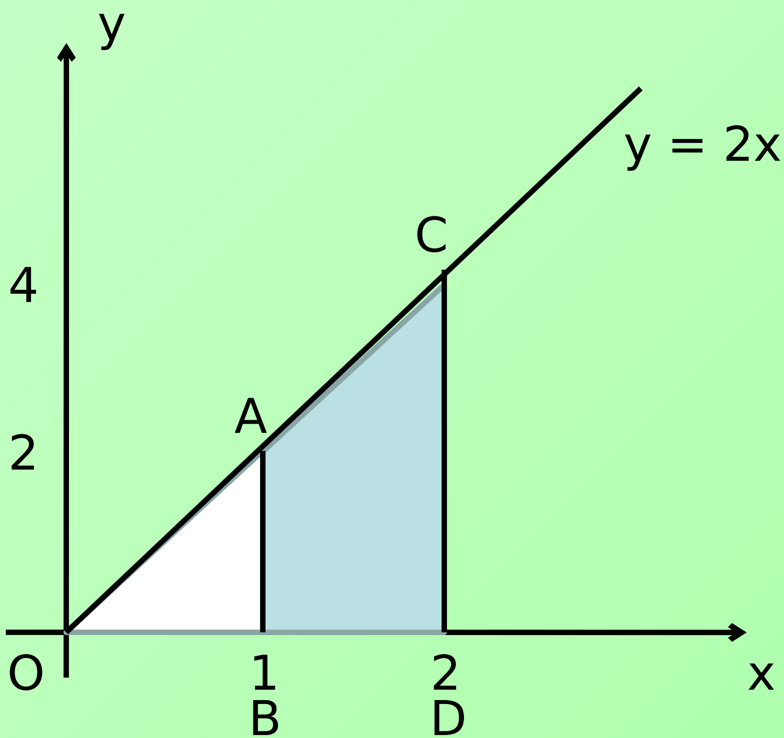
4.7 Area under a curve

The numerical value of a definite integral represents the area between the curve of , the -axis and the lines and .



4.7 Area under a curve

nd the blue shaded area:



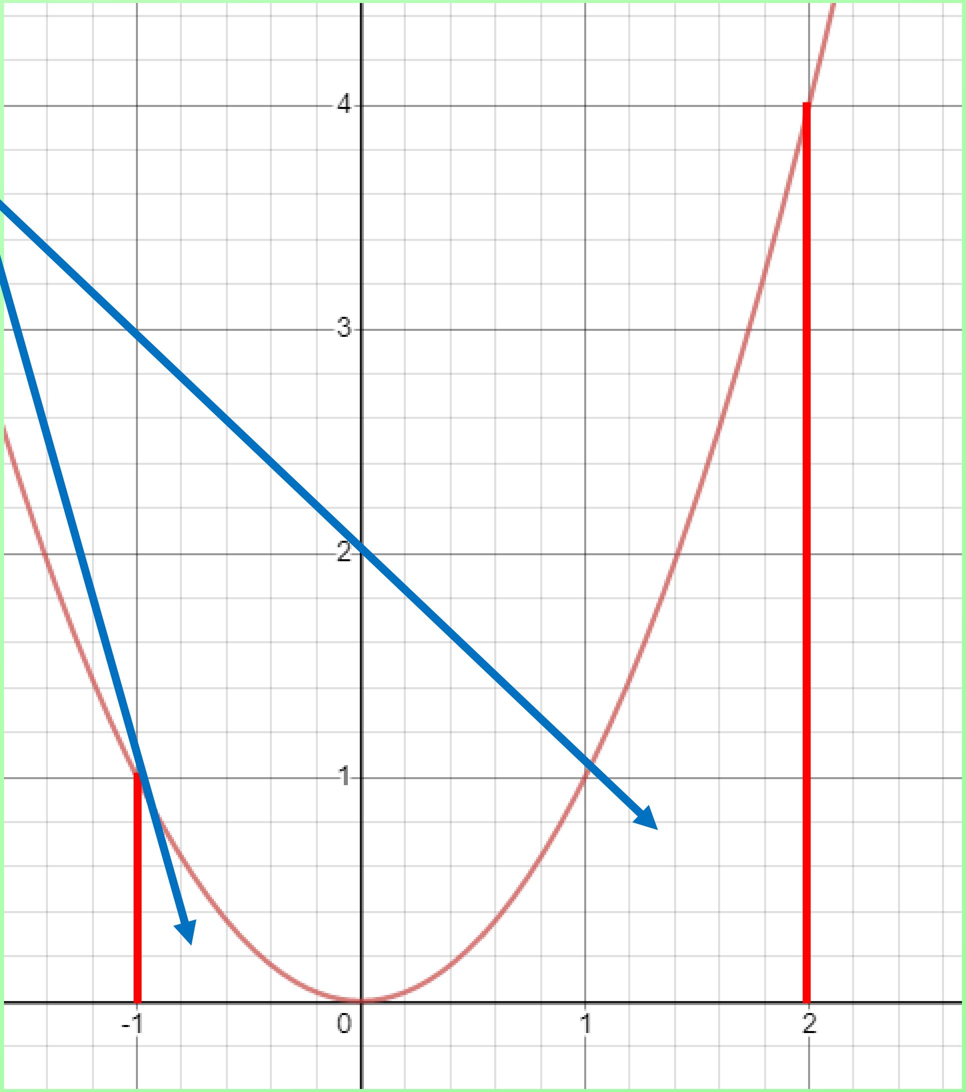
Compare with $\int_1^2 2x\,dx$

square units

rea of triangle OCD $= 0.5 \times 2 \times 4 = 4$
rea of triangle AOB $= 0.5 \times 1 \times 2 = 1$
herefore area of ABCD $= \mathbf{4 - 1 = 3}$

4.7 Area under a curve

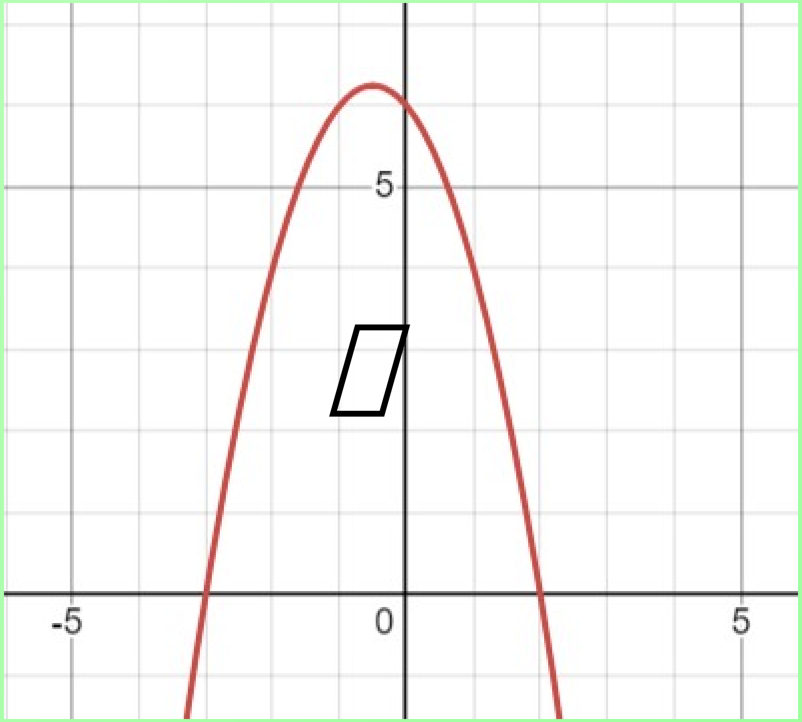
Example 4: Find the area bounded by the curve with equation, the -axis, and the lines and



4.7 Area under a curve

Example 5: Find the area of the finite region between the curve with equation and the -axis.

Sketch the graph first!



Tuesday, May 27, 2025

4.7 Area under a curve

Example 6: The area bounded by , the -axis and the lines and is . Find the possible values of where .

$$\int_1^4 \left(\frac{3}{7}x^2 + 2Ax^{-\frac{1}{2}}\right)dx = 5A^2$$

$$\left[\frac{1}{7}x^3 + 4Ax^{\frac{1}{2}}\right]_1^4 = 5A^2$$

$$\left(\frac{64}{7} + 8A\right) - \left(\frac{1}{7} + 4A\right) = 5A^2$$

Area Under Curves worksheet